Mathematics Test

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This test has been written for applicant wishing to pursue a postgraduate programme at the Department of Computer Science, Royal Holloway.

Our MSc programmes require good command on a few topics in mathematics. Although pre-sessional maths classes will run during Welcome Week, it is advisable that students revise these topics as indicated in the FAQ section of our web site. The questions below indicate what command of mathematics you should have to do well in the programmes. You can use these questions as a guidance for your revision.

1. Work out the following matrix products where possible:

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 \\ -1 & 2 & 1 \end{pmatrix} \ ;$$

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 2 \\ 3 & 1 \end{pmatrix} \ ;$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 2 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \ .$$

2. Let M be a symmetric positive definite matrix. What can you say about eigenvalues of M?

3. Let M be a square matrix such that $\det M = 0$. What can you say about eigenvalues of M?

4. You toss a coin twice. Which is more probable, getting the same outcome or different outcomes?

5. Let A and B be some events such that

$$\Pr(A) = 0.8 \ ;$$
$$\Pr(B) = 0.5 \ ;$$
$$\Pr(A \mid B) = 0.8 \ .$$

Calculate

$$\Pr(B \mid A) \ .$$

6. Let A and B be independent events such that
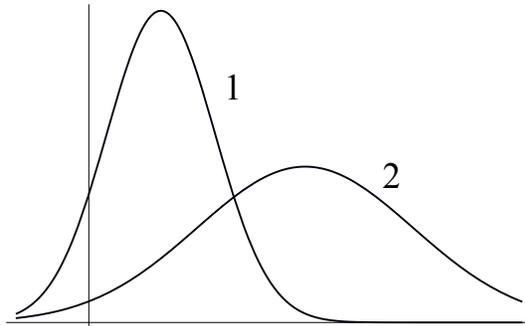
$$\Pr(A) = 0.8 \ ;$$
$$\Pr(B) = 0.5 \ .$$

Calculate

$$\Pr(B \mid A) \ .$$

7. Let $p(x)$ be the density function of the standard normal distribution $\mathcal{N}(0,1)$ (i.e., the mean equals 0 and the variance equals 1). Work out the integral

$$\int_{-\infty}^{0} p(x)dx \ .$$

8. The following graph shows the density functions of two normal distributions (drawn to the same scale). Which one has larger

   (a) mean;
   (b) variance?

9. The symmetric difference $A \triangle B$ of two sets A and B is the set containing those elements in either A or B, but not in both A and B. Answer the following questions:

   (a) Show the symmetric difference of the sets A and B on a Venn diagram.
   (b) Work out the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

(c) Express the symmetric difference $A \triangle B$ in terms of the basic set operations $\cup$, $\cap$, and $\setminus$.

10. Consider an undirected graph G with n vertices.

    (a) What is the number of edges in G if G is a tree?

    (b) What is the number of edges in G if each vertex of G has degree d?

    (c) What is the number of edges in G if G is complete?

    (d) What is the degree of each vertex in G if G is complete?

    (e) Suppose that all vertices in G have the same degree d. What is the minimum value of d if G is connected and $n > 2$?